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the equation of the pedal is at once seen to be

$$r = a + b \cos \theta,$$

which is the polar equation of the limaçon. Accordingly, the limaçon is the inverse of the reciprocal of the circle, that is, it is the inverse of a conic with respect to a focus.

The reciprocal of a conic with respect to a point on the curve is a parabola; with respect to a point outside the curve (that is one from which real tangents can be drawn) the reciprocal is an hyperbola; and for a point within the curve, it is an ellipse.

Accordingly, the pedal with respect to a point on the curve, like the inverse of the parabola, is cuspidal; for a point without, it is crunodal, like the inverse of an hyperbola; and for a point within, it is acnodal, like the inverse of an ellipse. On the other hand, the pedal to the parabola, like the inverse with respect to a point on the curve, is a cubic, while the pedals of the other conics are quartics.

THE DIFFERENTIAL EQUATIONS OF PROBLEM 165.

BY PROF. A. HALL, NAVAL OBS., WASHINGTON, D. C.

SINCE the differential equations of this problem are those which furnish the theory of the pure elliptic motion of the planets they have been very completely discussed, and it seems worth while to notice a few of the properties that have been deduced. First, we may show without any integration that the planet moves in a plane. If we consider three successive positions of the planet whose coordinates are x , $x+dx$, and $x+2dx+d^2x$, with similar values for y and z ; and form the determinant of the third order whose elements are these coordinates, by means of the given differential equations this determinant reduces to zero, which is the condition that the three points are in a plane.

One of the most elegant discussions of the equations is that given by Laplace, *Mec. Cel.*, Book II. arts. 17 and 18. Laplace first notices that each of the equations may be written in the form

$$d.\left(r^3 \cdot \frac{d^2x}{dt^2}\right) + m \cdot dx = 0,$$

and then shows that we have also the similar form

$$d.\left(r^3 \cdot \frac{d^2r}{dt^2}\right) + m \cdot dr = 0. \tag{1}$$

From this last form he deduces the linear equation for r ,

$$r = \frac{h^2}{m} + \lambda x + \gamma y,$$

h , λ and γ being constants. In his *Theoria Motus*, art. 3, Gauss uses this equation in the form

$$r + \alpha x + \beta y = \gamma,$$

and has the remark “et quidem γ quantitatem natura sua semper positivam;” which from Laplace’s form is evidently correct.

If we integrate equation (1) we have

$$\frac{d^2 r}{dt^2} + \frac{m}{r^2} + \frac{k}{r^3} = 0,$$

the constant k having a value different from zero. Hesse gives the following interpretation to this equation: If one lived on the radius vector of a planet he would not be able to explain the motion of the planet by means of the law of gravitation, but must regard the inverse third power of the distance.

TWELVE ORIGINAL PROBLEMS.

BY PLINY EARLE CHASE, LL. D., PROF. OF PHILOS. IN HAVERFORD COL.

1. LET f represent any central force whatever, varying inversely as the square of the distance, r being the radius of a perpetual circular oscillation produced by the force. Required the mean velocity of a synchronous radial oscillation.

2. If material particles under the influence of f were constrained to move in various orbits, (circular, elliptic, parabolic, and rectilinear,) collisions near the centre would produce a nucleus, which would rotate, on account of the resultant of such portions of the orbital forces as were not otherwise represented. Required the law of varying velocity, in terms of r , under nuclear contraction or expansion.

3. If a viscous nucleus were accompanied by a non-viscous, elastic, atmosphere, rotating with the nucleus on account of central pressure, what formula would express the limit of possible atmosphere, in terms of f , r , and v , r being the nuclear radius and v the velocity of equatorial rotation?

4. If such a nucleus and atmosphere are simultaneously expanding or contracting, express the varying ratio of the nuclear radius (r) in terms of the atmospheric radius (r').

5. Give an expression for the common tangential velocity towards or from which the circular-orbital velocity (v') and the equatorial-rotation velocity (v) both tend.